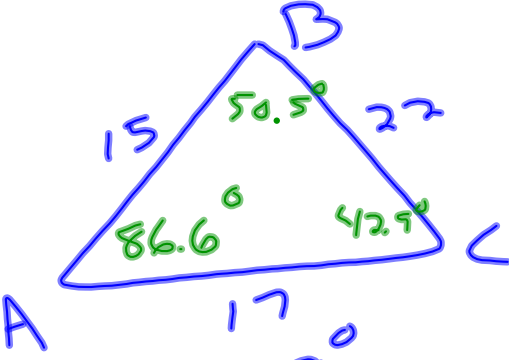6-1 - 6-2   The Law of Sines and Law of Cosines

Ex. 1   Solve the triangle given: $a = 22, \quad b = 17, \quad c = 15$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$15^2 = 22^2 + 17^2 - 2 \cdot 22 \cdot 17 \cdot \cos C$$

$$225 = 484 + 289 - 748 \cos C$$
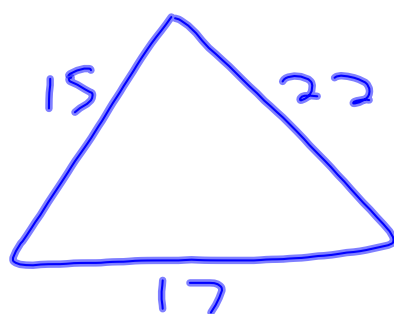
$$225 = 773 - 748 \cos C$$

$$50.5° \approx \cos C$$

$$\approx C$$

$$C \approx 42.9°$$

$$\frac{\sin 42.9°}{15} = \frac{\sin B}{17}$$

$$B \approx 50.5°$$

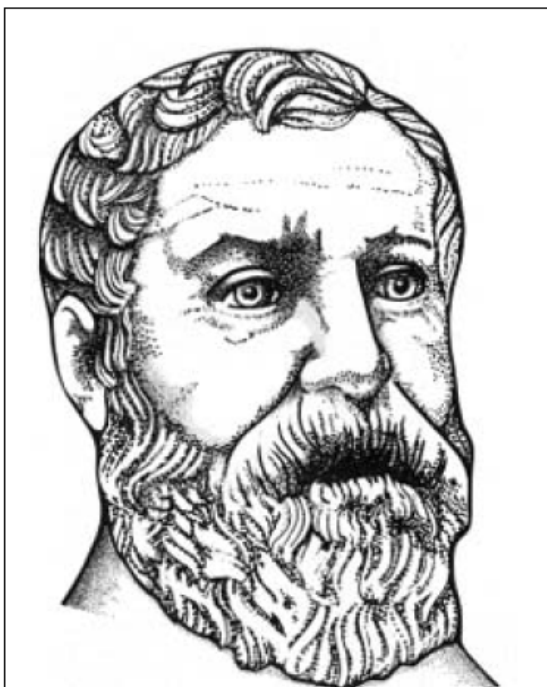Ex. 2   Find the area of the triangle in the previous example.

$$s = \frac{a + b + c}{2}$$

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\sqrt{27(5)(10)(12)}$$
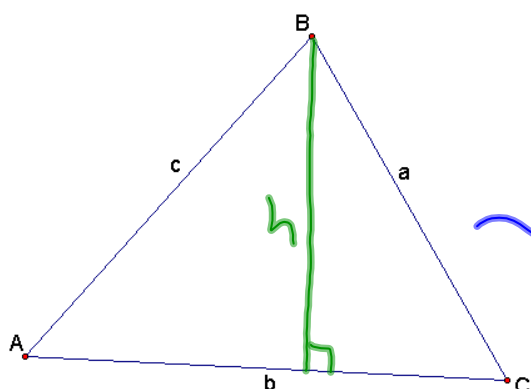
$$\approx 127.28$$

Heron
10 - 70 A.D.

Heron
10-70 A.D Alexandria, Egypt

Heron, also known as Hero, of Alexandria, Roman Egypt was a Greek engineer and geometer. He was an atomist, people who believe the universe to be made of tiny particles called atoms. He taught at the Musaeum and Library of Alexandria thus most of his writings appear as lecture notes in mathematics, mechanics, physics, and pneumatics. Heron was mainly known for his mechanical inventions, contributing from the theater to the military. Some inventions of his include: the Aeolipile (the first steam engine), first vending machine, and mechanical theater props. Some of his major works include his study of light Optics (concluding that the shortest path traveled is a straight line), Heron's Formula, and his best work *Metrica*. Heron was also known to mystify people by creating devices that would make events occur as if the gods themselves were doing them.

Major Works:
Heron's Formula
Metrica

Proof of the law of sines

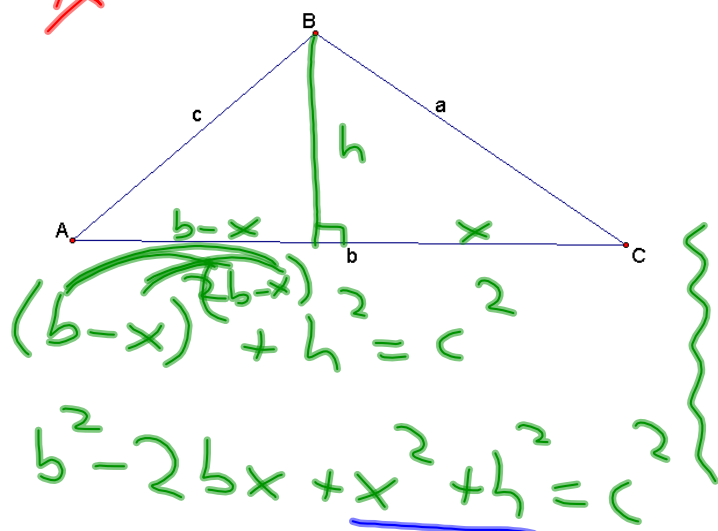$$\sin A = \frac{h}{c}$$

$$c \sin A = h$$

$$\sin C = \frac{h}{a}$$

$$a \sin C = h$$

$$\frac{c \sin A}{ac} = \frac{a \sin C}{ac}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

Proof of the Law of Cosines

$$(b-x)^2 + h^2 = c^2$$

$$b^2 - 2bx + x^2 + h^2 = c^2$$

$$b^2 - 2bx + a^2 = c^2$$

$$b^2 - 2ba\cos C + a^2 = c^2$$

$$x^2 + h^2 = a^2$$

$$\cos C = \frac{x}{a}$$

$$a\cos C = x$$

$$\rightarrow \boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

Homework

p.414 #1-7, 19-24

p.421 #1-19 odds, 27-36